**Homework 7**

**P7.1.2** A parallel plate capacitor consists of two plates, each being  square, separated by 1 mm (Figure P7.1.2). Half the space between the plates, in the vertical direction, is filled by a dielectric of relative permittivity 10. Determine the capacitance, neglecting edge effects. Show that the capacitance can be considered as that of two capacitances in series.



**Solution:** Referring to Figure 7.1.1c, the charge enclosed by the cylinder is *σΔA*, where *σ* is the charge density *q*/A. Hence, *D* = *σ* and is continuous across the separation between the dielectric material and air. *ξi* in the region occupied by the dielectric material is *D*/*ε*0*εr*, where *ε*0 = 8.85×10-12 F/m and *εr* = 10. *ξa* in the air region is *D*/*ε*0. The voltage *V* between the plates is *ξi**di* + *ξa**da*, where *di* = *da* = 0.5 mm. Substituting *D* = *q*/*A*,  It follows that *C* = *q*/*V* = =  F ≡ 40.23 pF.

The structure is equivalent to two capacitors in series, one filled with air, the other with dielectric. The capacitance of the air-filled capacitor is *ε*0*A*/*d* =  F ≡ 44.25 pF. The capacitance of the dielectric-filled capacitor is 10 times as much, or 442.5 pF. The equivalent series capacitance is  pF.



**P7.1.8** The current in a 2 μF capacitor is shown in Figure P7.1.8 as a function of time. Determine the charge on the capacitor at *t* = 3.4 s.

**Solution:** The charge at *t* > 3 s is the net area under the curve, which gives *q* = 2×3 – 1×(*t* –3) = (9 – *t*) = 5.6 C.

**P7.1.11** The voltage applied to an initially uncharged 0.1 μF capacitor is the first half-cycle of the waveform 10sin500*t* V, where  is in seconds, Derive the expressions for the capacitor current, the energy stored in the capacitor, and the instantaneous power input to the capacitor. Sketch the time variation of these quantities.

**Solution:** *ω* = 500 rad/s, *T* = 2*π*/500 = 4*π* ms, and the duration of a half-cycle is 2*π* ms. *q* = *Cv* = 0.1×10-6×10sin(500*t*) C, where *t* is in s, or *q* = sin(0.5*t*) μC, ms, and *q* = 0 elsewhere.



*i* == 0.5cos(0.5*t*) mA, ms, and *i* = 0 elsewhere.

*w* = =



5sin2(0.5*t*) μJ,



ms, and *i* = 0 elsewhere.

*p* = 10sin(500*t*) ×0.5×cos(500*t*) = 5sin(500*t*)cos(500*t*) = 2.5sin(1000*t*) mW, and *p* = 0, elsewhere.

**P7.1.13** The triangular voltage pulse of Figure P7.1.13 is applied to a  capacitor that is initially uncharged. Plot as function of time: (a) the charge on the capacitor; (b) the energy stored in the capacitor, (c) the instantaneous power input to the capacitor.



**Solution:** *v* = V, s; *v* =V, s; *v* = 0, s.

(a) *q* =μC, s;



=μC, s;

= 0, s.

(b) *w* = μJ,



s;

=μJ,

s;

= 0, s.

(c) *i* = μA, s;



=μA, s;

= 0, s, where μA may also be expressed as 1 μC/min.

(d) *p* = μW, s; = μW, s;



= 0, s, where

μW may also be expressed as 10 μJ/min.

It is seen that . Thus μJ, s. At *t* = 60 s, *w* = 5 μJ. For s  μJ.

**P7.2.3** A voltage pulse of amplitude 100 μV and 200 ms duration is applied at *t* = 0 to a 2 μH inductor. Express the inductor current as a function of time, assuming (a) A the inductor is initially uncharged; (b) the inductor current is initially -10 A.

**Solution:** (a) ,  A, 0 ≤ *t* ≤ 200 ms. For *t* ≥ 200 ms, *i* = 50×200×10-3 = 10 A.

(b)  A, 0 ≤ *t* ≤ 200 ms. For *t* ≥ 200 ms, *i* = 50×200×10-3 = 0.

**P7.2.10** The triangular current pulse of Figure P7.2.10 is applied to a 0.1μH inductor that is initially uncharged. Plot as a function of time: (a) the flux linkage in the inductor; (b) the energy stored in the inductor, (c) the instantaneous power input to the inductor.



**Solution** *i* = A, s;

=A, s;

= 0, s.



(a) *λ* =μWb-T, s;

=μWb-T, s = 0, s.

(b) *w* = μJ, s;



=μJ, s;

= 0, s.

(c) *v* = μV, s;



*v* =μv, s;

*v* = 0, s,

where μV may also be expressed as 1 μWb-turns/min.

(d) *p* = μW, s;



=  μW, s;

= 0, s, where μW may also be expressed as 10 μJ/min.

t is seen that . Thus μJ, s. At *t* = 60 s, *w* = 5 μJ. For s  μJ.

**P7.2.13** The flux linkage in a 0.1 μH inductor varies with time as shown in Figure P7.2.13. Determine the average current through the inductor.



**Solution:** Average flux linkage is net area divided by the duration. The net area is  Wb-Tmin. The average flux linkage is 6×10-6/6 =

10-6 Wb-T. the average current is

10-6/(0.1×10-6) = 10 A.

**P7.2.17** Determine *Ix* in Figure P7.2.17, assuming a dc steady state, all resistances are in ohms, all inductances are 1 H, and all capacitances are 1 F.



**Solution:** When inductors are replaced by short circuits and the capacitors by open circuits, the circuit is as shown. When the 5 A source is applied alone, the circuit is as shown. 8||8 = 4 Ω, 4||12 = 3 Ω, 3 + 2 = 5 Ω. Hence, 25/7 A.

When the 100 V source is applied alone, the circuit is as shown. The upper resistances are 8||8||4 = 2 Ω. The current in the 6 Ω resistor is 100/14 = 50/7 A and  A.



When the 50 V source is applied alone, the circuit is as shown. The upper resistances are 8||8||4 = 2 Ω. The current in the 4 Ω resistor is 50/14 = 25/7 A and  A. It follows that  A

